Question 2 **(5 marks)**

(a) Determine $\frac{d}{dx}(2xe^{2x})$. (2 marks)

(b) Use your answer in part (a) to determine $\int 4xe^{2x}\,dx$. (3 marks)

Question 3 **(7 marks)**

Consider the function $f(x) = \frac{(x-1)^2}{e^x}$.

(a) Show that the first derivative is $f'(x) = \frac{-x^2 + 4x - 3}{e^x}$. (2 marks)

(b) Use your result from part (a) to explain why there are stationary points at $x = 1$ and $x = 3$. (2 marks)

Question 3 (continued)

It can be shown that the second derivative is $f''(x) = \dfrac{x^2 - 6x + 7}{e^x}$.

(c) Use the second derivative to describe the type of stationary points at $x = 1$ and $x = 3$. (3 marks)

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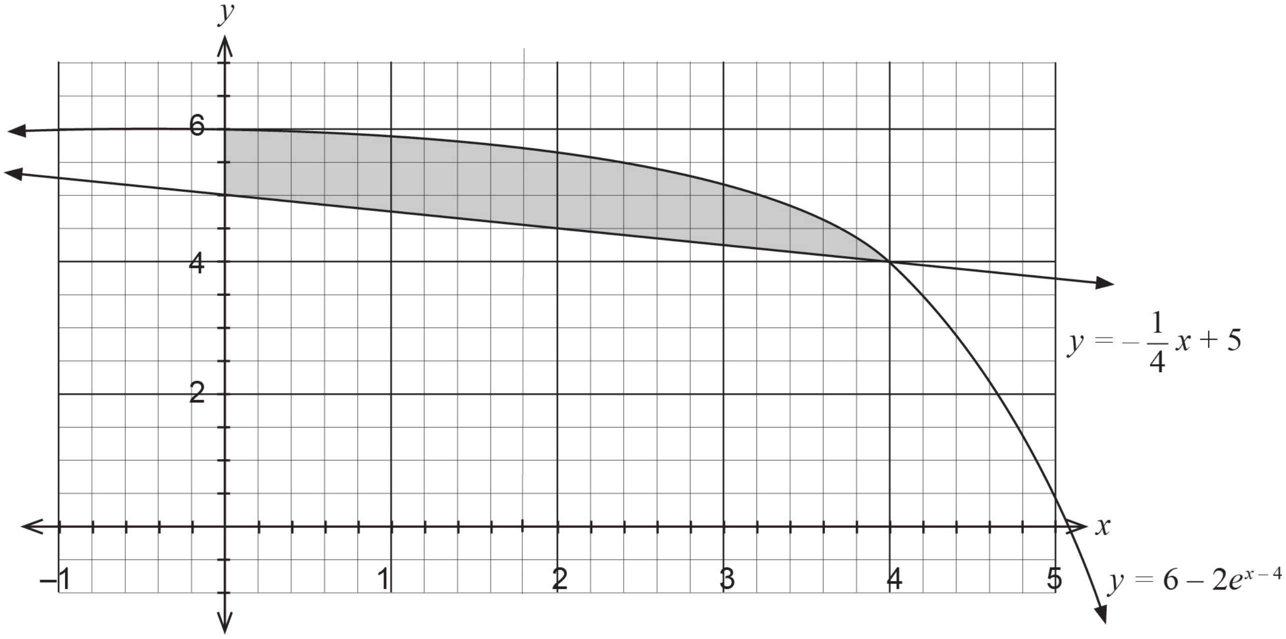
Question 6 **(4 marks)**

The graphs $y = 6 - 2e^{x-4}$ and $y = -\frac{1}{4}x + 5$ intersect at $x = 4$ for $x \geq 0$.



Determine the exact area between $y = 6 - 2e^{x-4}$, $y = -\frac{1}{4}x + 5$ and the y axis for $x \geq 0$.

Question 15 **(10 marks)**

The volume $V(h)$ in cubic metres of a liquid in a large vessel depends on the height h (metres) of the liquid in the vessel and is given by

$$V(h) = \int_0^h e^{\left(-\frac{x^2}{100}\right)} dx, \quad 0 \le h \le 15.$$

(a) Determine $\frac{dV}{dh}$ when the height is 0.5 m. (2 marks)

(b) What is the meaning of your answer to part (a)? (1 mark)

(c) The height of the liquid depends on time t (seconds) as follows:

$$h(t) = 3t^2 - t + 4, \quad t \ge 0.$$

(i) Determine $\frac{dh}{dt}$ when the height is 6 m. (2 marks)

(ii) Use the chain rule to determine $\frac{dV}{dt}$ when the height is 6 m. (2 marks)

(iii) Given the volume of the liquid at 2 seconds is 8.439 m^3, use the incremental formula to estimate the volume 0.1 second later. (3 marks)

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Question 16 **(8 marks)**

A group of biologists has decided that colonies of a native Australian animal are in danger if their populations are less than 1000. One such colony had a population of 2300 at the start of 2011. The population was growing continuously such that $P = P_0 e^{0.065t}$ where P is the number of animals in the colony t years after the start of 2011.

(a) Determine, to the nearest 10 animals, the population of the colony at the start of 2014. (2 marks)

(b) Determine the rate of change of the colony's population when $t = 2.5$ years. (2 marks)

(c) At the beginning of 2017, a disease caused the colony's population to decrease continuously at the rate of 8.25% of the population per year. If this rate continues, when will the colony become 'in danger'? Give your answer to the nearest month. (4 marks)

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Question 9 **(8 marks)**

The concentration, C, of a drug in the blood of a patient t hours after the initial dose can be modelled by the equation below.

$$C = 4e^{-0.05t} \text{ mg/L}$$

Patients requiring this drug are said to be in crisis if the concentration of the drug in their blood falls below 2.5 mg/L.

A patient is given a dose of the drug at 9 am.

(a) What was the concentration in the patient's blood immediately following the initial dose? (1 mark)

(b) What is the concentration of the drug in the patient's blood at 11.30 am? (2 marks)

(c) Find the rate of change of C at 1 pm. (2 marks)

(d) What is the latest time the patient can receive another dose of the drug if they are to avoid being in crisis? (3 marks)

Question 14 **(5 marks)**

(a) The table below examines the values of $\frac{a^h - 1}{h}$ for various values of a as h approaches zero. Complete the table, rounding your values to five decimal places. (2 marks)

h	$a = 2.60$	$a = 2.70$	$a = 2.72$	$a = 2.80$
0.1	1.00265		1.05241	1.08449
0.001	0.95597	0.99375		
0.00001	0.95552			1.02962

It can be shown that $\frac{d}{dx}(a^x) = a^x \lim_{h \to 0}\left(\frac{a^h - 1}{h}\right)$.

(b) What is the exact value of a for which $\frac{d}{dx}(a^x) = a^x$? Explain how the above definition and the table in part (a) support your answer. (3 marks)

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